Using non-standard student solutions to probe what it means to solve linear equations in school

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Background - The mathematics of solving equations : $ax + b = cx + d$

Canonical method

$2(x - 4) = -2x + 6$

$2x – 8 = - 2x + 6$

$2x + 2x = 6 + 8$

$4x = 14$

$x = 3.5$

“Divide first” method

$2(x - 4) = -2x + 6$

$x – 4 = - x + 3$

$2x = 7$

$x = 3.5$

Other methods

False Position

Tables

Graphs

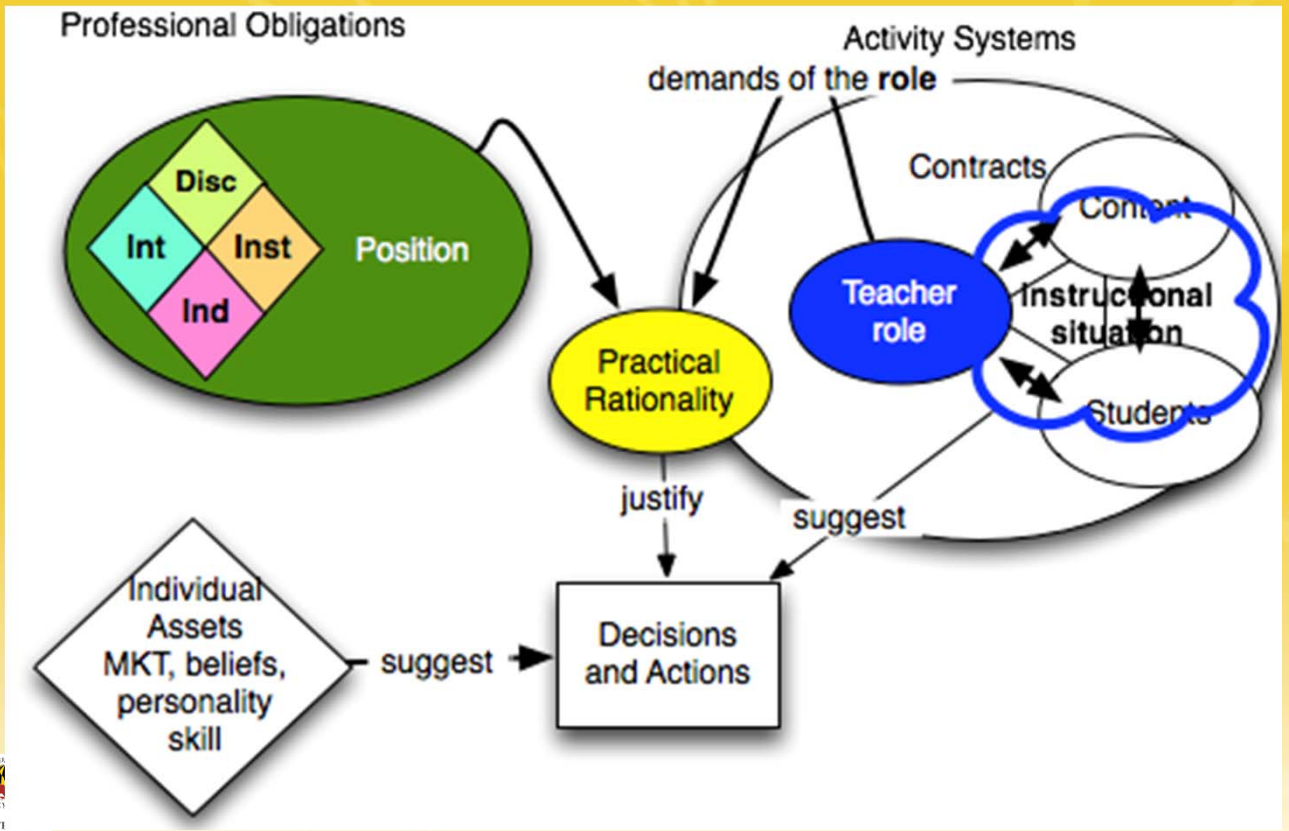
….

Research Questions

- Do teachers indicate that when teaching students to solve equations their responsibility is to see that students have learned the canonical method?
  - What do teachers view as appropriate responses to non-standard student work?

Practical rationality

Algebra teachers are a group of practitioners, who's work can be characterized by a shared *practical rationality:* a set of actions, discourses, perceptions, intuitions, values and principles that underline their work and which are held tacitly as form of social knowledge, Linde (2000).



Why are only certain types of solutions to linear equations common in school?

Instructional situations as an element of practical rationality

- Norms regulate a market-like interaction between teacher and student around student work (Bourdieu, 1998; Herbst, 2006).
- We use methods modeled on breaching experiments from sociology (Garfinkel, 1967) to study the complexity of norms of social interaction in the classroom.
- In this paper, we are exploring the norms of the instructional situation for solving linear equations.

Data collection and analysis

- 45 teachers from very large suburban school districts in one state. 13 male, 32 female. Diverse teaching experience: range 2 to 35 years: $\bar{x} = 17.56$. All taught Algebra 1 in the last 4 years.
- Completed multiple instruments over one, two or three full days' workshops.
- Online rich-media questionnaires:
  - Closed-ended verbal items
  - Story- based instrument
- Data analysis involved both qualitative and quantitative techniques.

Quantitative:

- Descriptive statistics.
- Correlational analyses.
- Cluster analysis.

Qualitative:

- Exploratory analysis with partially pre-defined categories (obligations and norms).
- Systemic Functional Linguistics (SFL) methods.

Mixed methods

- Cross referencing

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Closed-ended verbal items

Structure of 5 items

When	**assigning**	linear equations, how appropriate is it for an Algebra teacher to	expect that students will solve it by **operating on both sides** of a linear equation?
	practicing solving		guide students to use **one general strategy** for solving equations?
	asking students to **present** solutions to		**call on** only those students who solved the equation by operating on both sides?
	evaluating students' solutions of		**accept only solutions** that involve operating on both sides of equation?
	evaluating students' solutions of		**reject graphical**, **rather than symbolical**, solutions to an equation?

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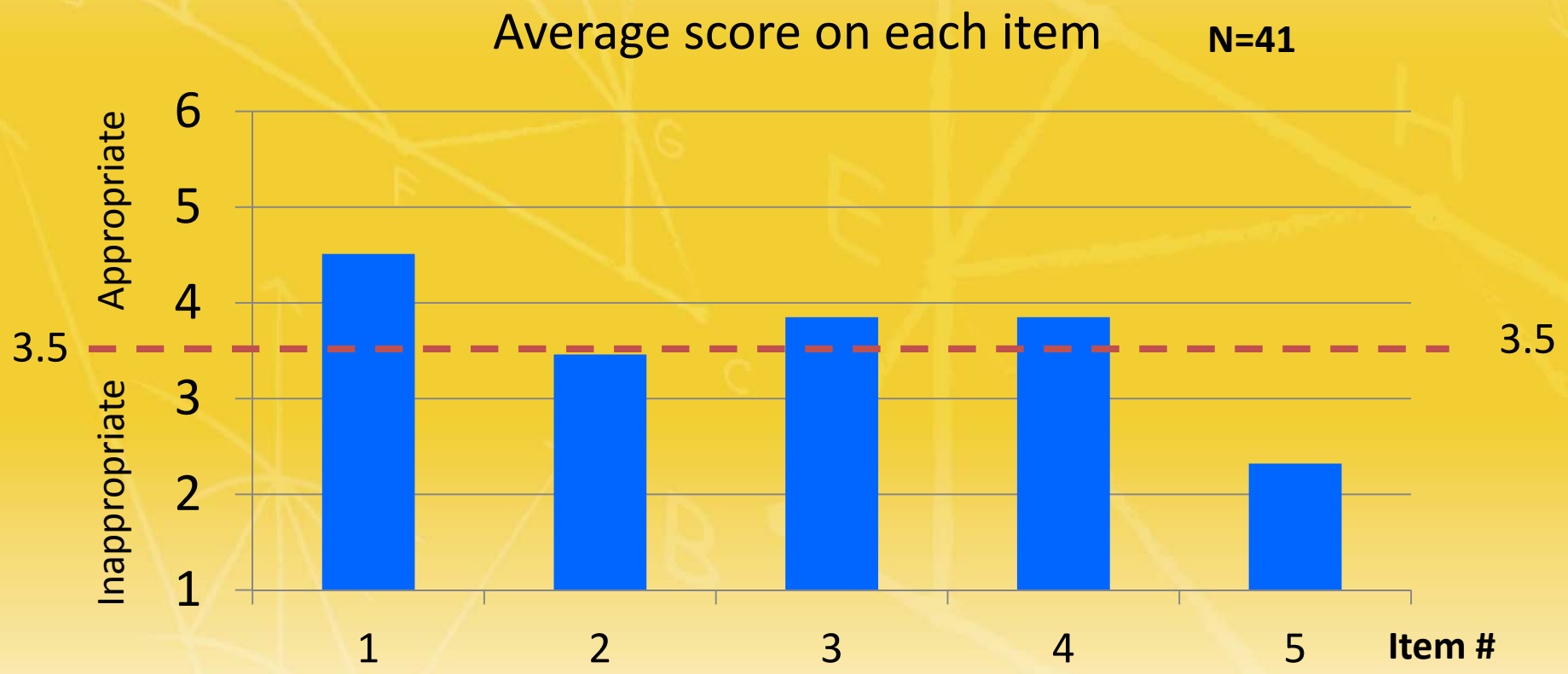
Closed-ended verbal items: Judgments of appropriate-ness of preference / no preference towards canonical solutions vs. other solutions.

Likert scale 1-6 (1: very inappropriate, 6 – very appropriate)



Across 5 questions, the participants slightly lean towards "appropriate to insist on canonical solutions", with an exception of question #5.

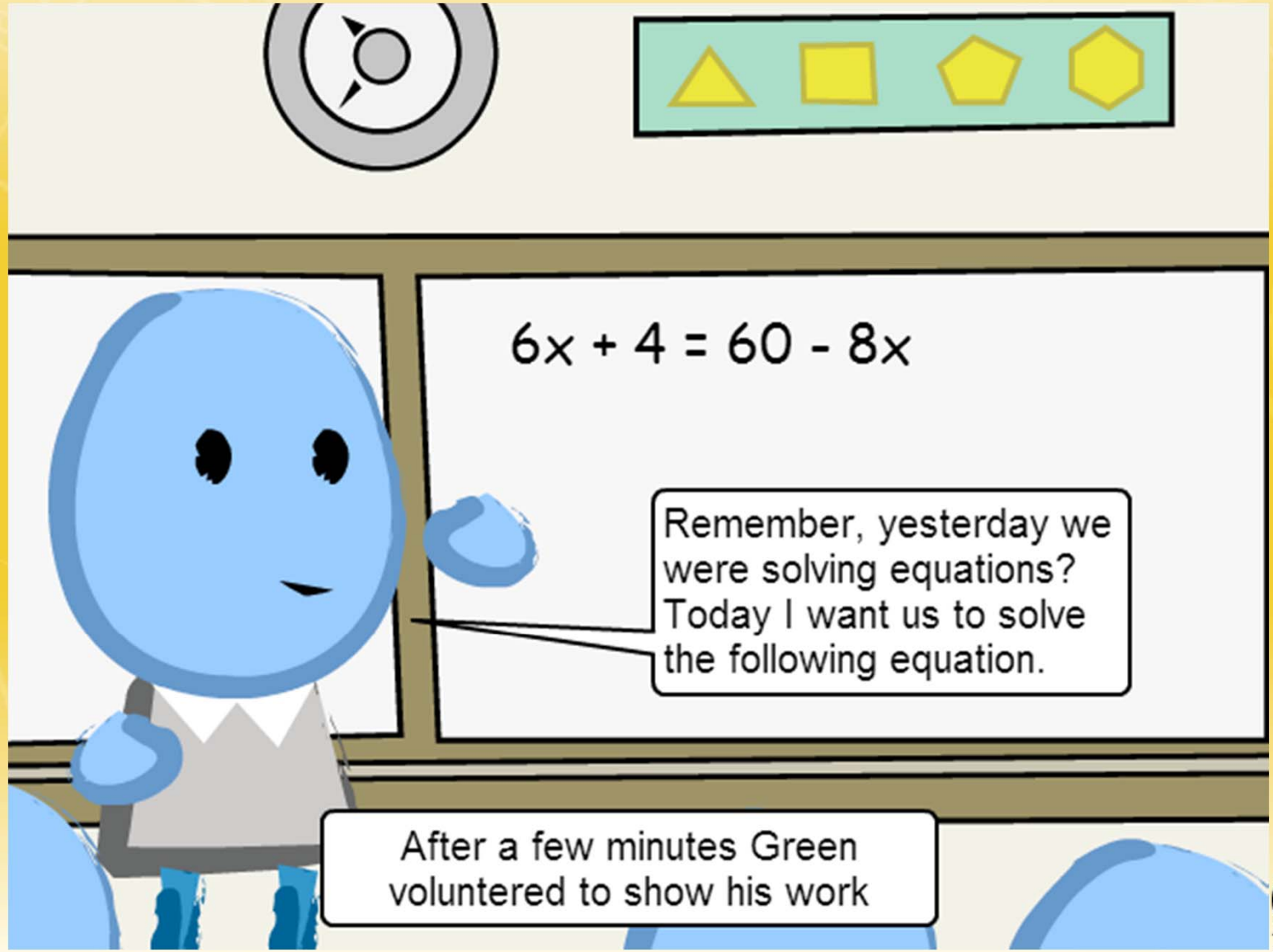
Story-based instrument

- Stories that represent classroom situations in which a student volunteers a non-standard solution to linear equation (e.g. divide first).
- The student breaches a hypothesized norm for solving linear equations, they do not use the canonical method.
- The teacher prefers canonical solutions over another types of solutions. (A "Rejection story": Student reasoning is rejected.)

Questions asked regarding the story

Question	Type
What did you see happening in this scenario?	open ended
Viewed as facilitation of work on solving of equations, **how appropriate was the teacher's work in this scenario**?	Likert (1 - 6) 1: Very Inappropriate 6: Very Appropriate
Please explain your reasoning.	open ended
Viewed as facilitation of work on solving of equations, **how appropriate was the teacher's response to Yellow's solution**?	Likert (1 - 6) 1: Very Inappropriate 6: Very Appropriate
Please explain your reasoning.	open ended

The story item: “Divide first”



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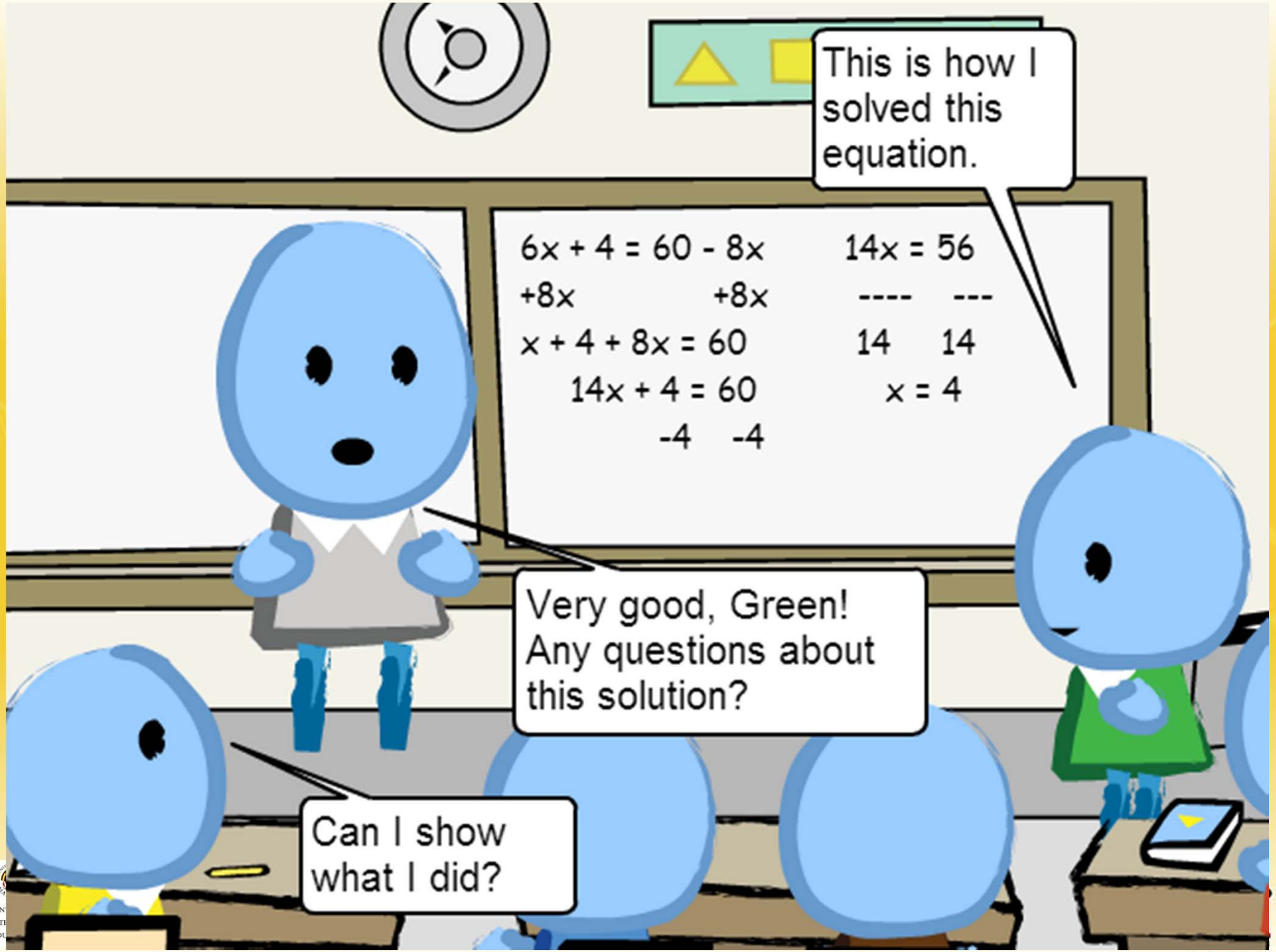
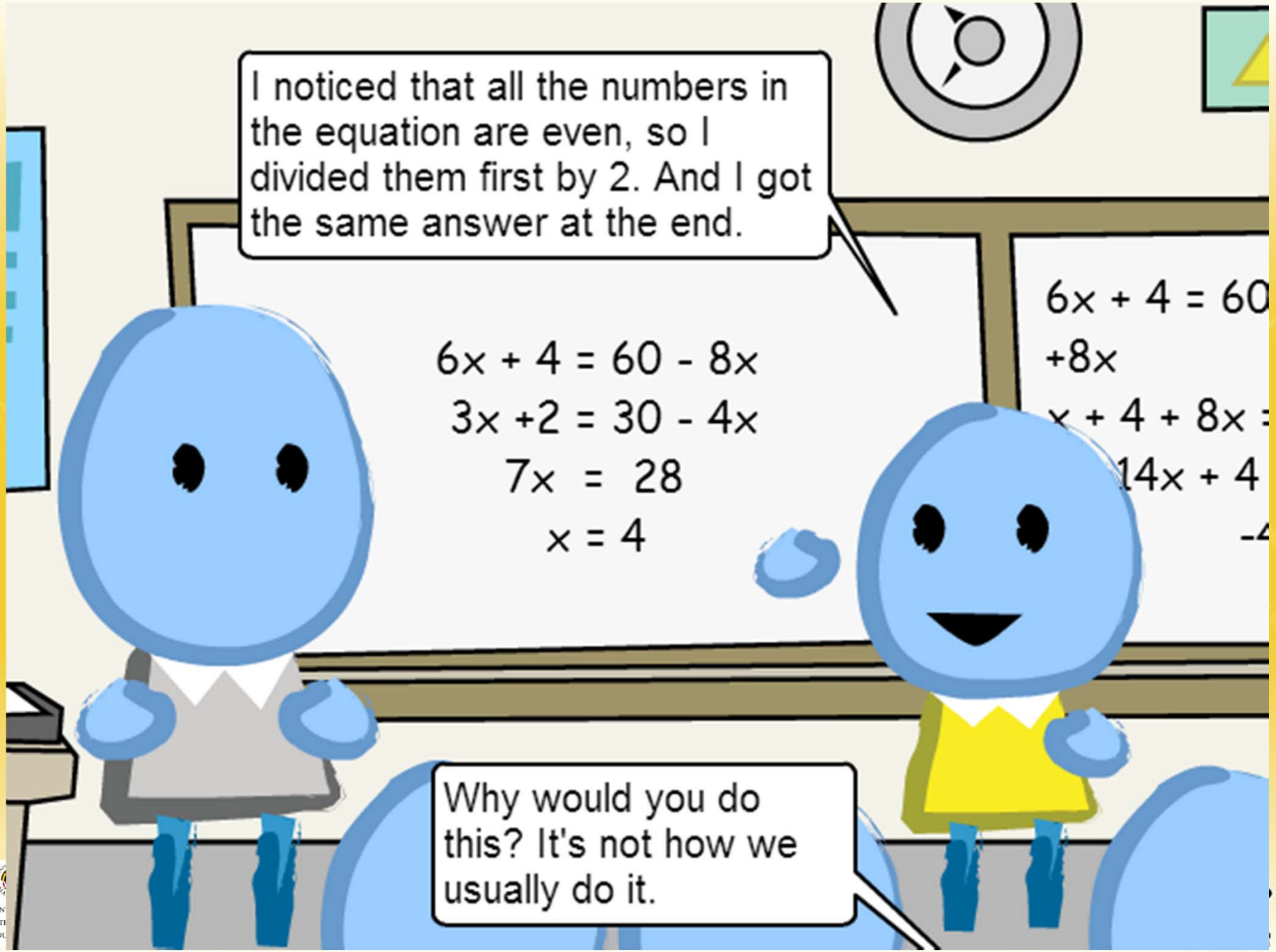
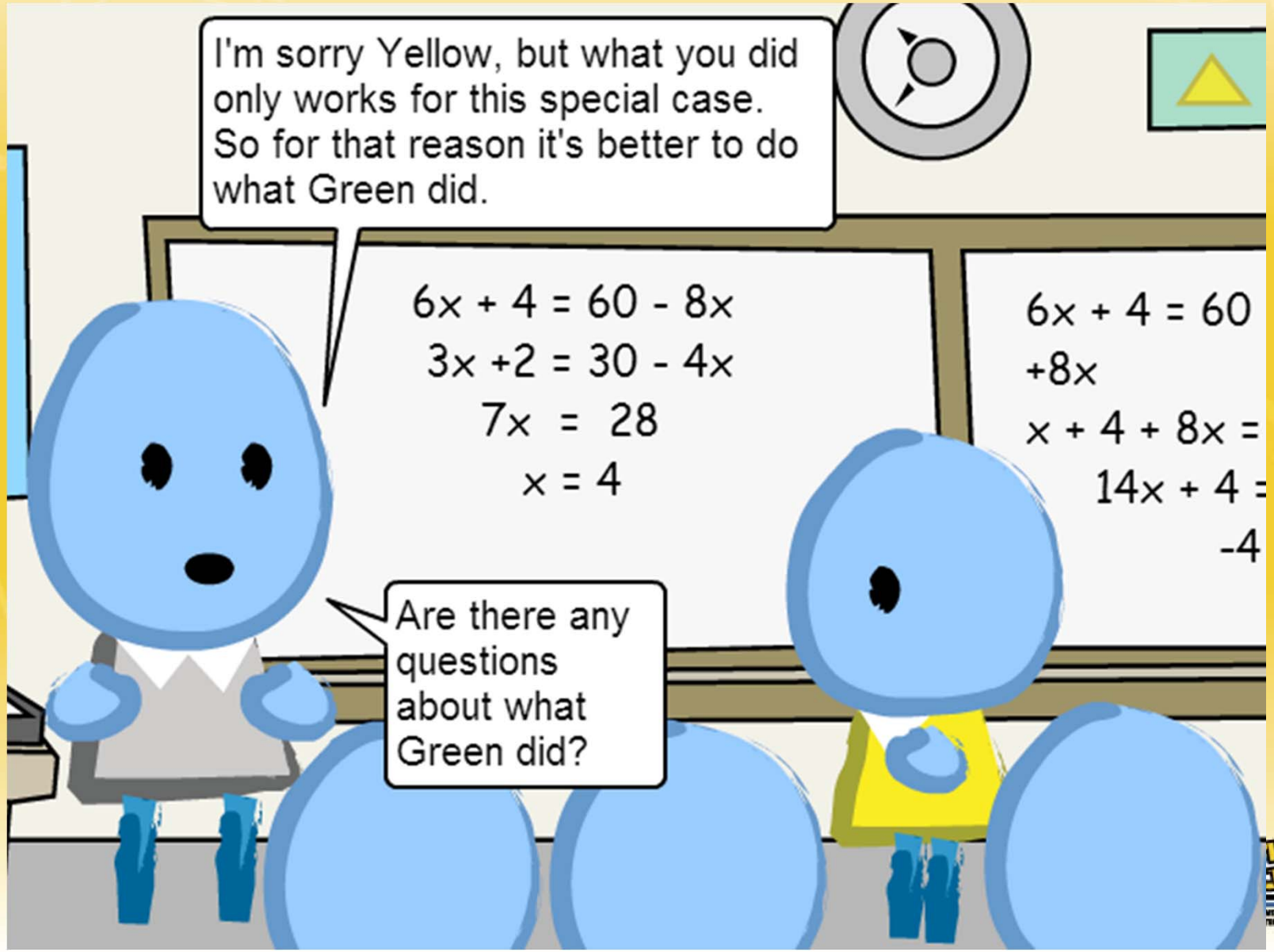

This is how I solved this equation.
6x + 4 = 60 - 8x
+8x +8x
x + 4 + 8x = 60
14x + 4 = 60
-4 -4
14x = 56
---- ---
14 14
x = 4
Very good, Green! Any questions about this solution?
Can I show what I did?


I noticed that all the numbers in the equation are even, so I divided them first by 2. And I got the same answer at the end.
6x + 4 = 60 - 8x
3x +2 = 30 - 4x
7x = 28
x = 4
6x + 4 = 60
+8x
Why would you do this? It's not how we usually do it.

The teacher shows preference for a canonical solution



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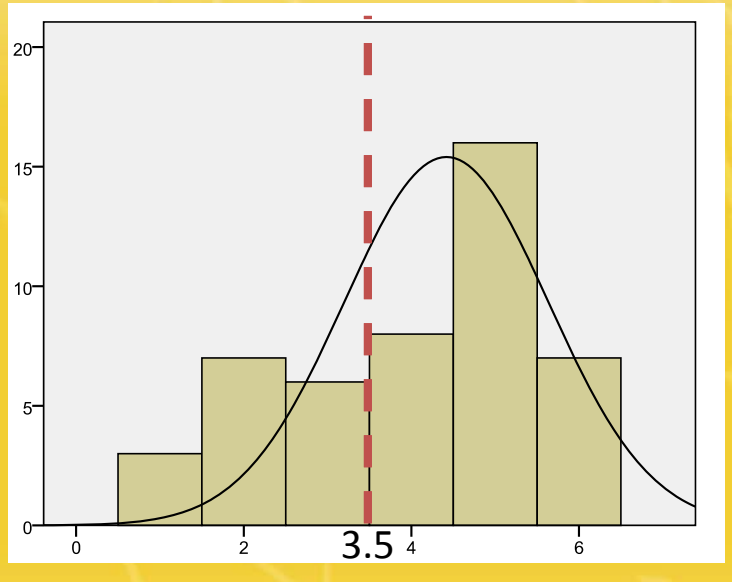
Judgments of the appropriate-ness of the teacher's preference of the canonical method over Yellow's method.

N=47

How appropriate was the teacher's work in this scenario?

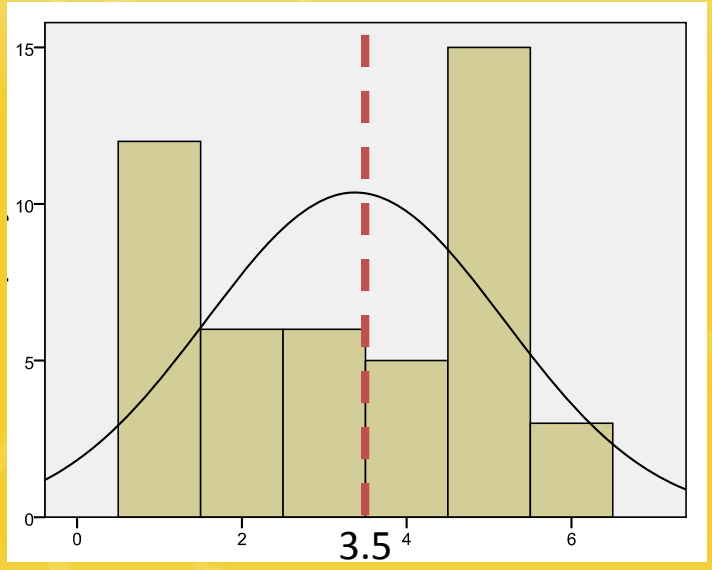


Inappropriate (1, 2, 3) 34% $\bar{X} = 1.8$

Appropriate (4, 5, 6) 66% $\bar{X}$ =4.97

$\bar{X}$= 4.02, Std.Dev. = 1.511

How appropriate was the teacher's response to Yellow's solution?



Inappropriate (1, 2, 3) 51% $\bar{X} = 1.75$

Appropriate (4, 5, 6) 49% $\bar{X} = 4.91$

$\bar{X}$ = 3.30, Std.Dev. = 1.756

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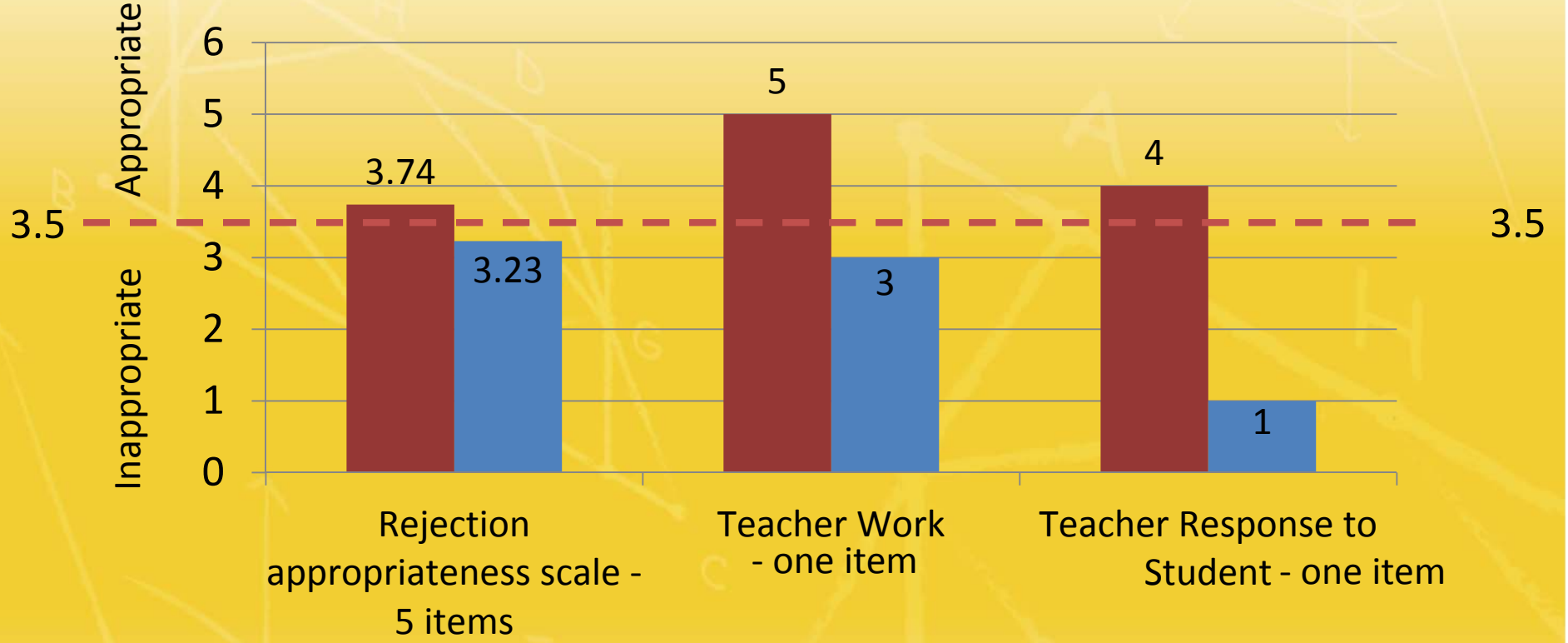
Cluster analysis across the two types of instruments: Two clusters

N = 34



Cluster 1: N=21

- Scored **higher** on every tested variable.
- Appropriate to **prefer canonical solutions** over other solutions (e.g. divide first).

Cluster 2: N= 13

- Scored **lower** on every tested variable.
- **Inappropriate to prefer canonical solutions** over other solutions (e.g. divide first).

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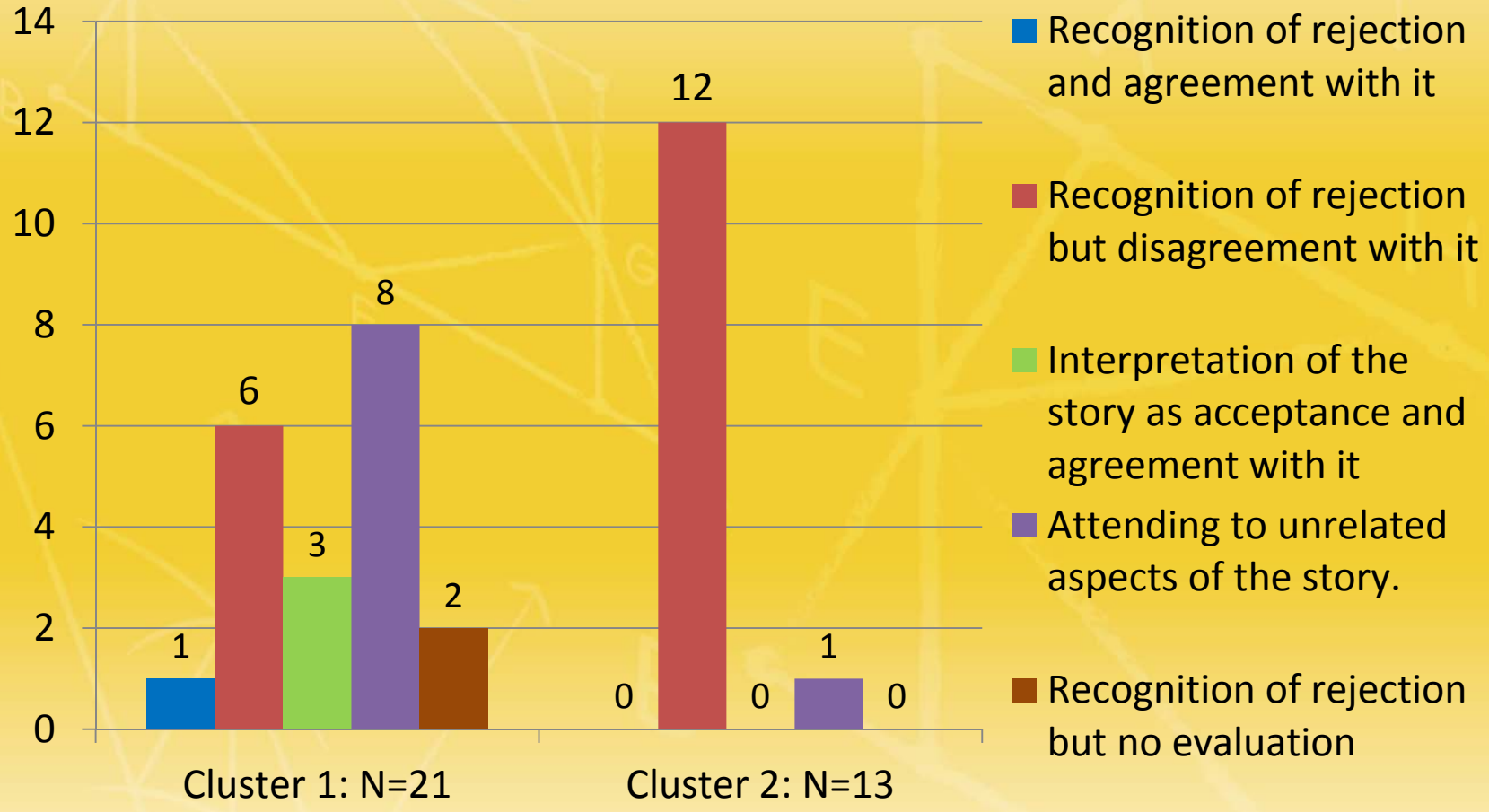
Story based instrument: Analysis of open-ended responses

- We used Systemic Functional Linguistics (SFL) Methods (Eggins, 2004) to analyze open-ended responses.
- We coded for **recognition** of represented teacher's action as "rejection" of "divide-first" non-canonical solution method.
- We coded for **appraisals** related to the norm:
  - Positive appraisals
  - Negative appraisals

Appraisals of teacher's actions	Recognition of "Rejection of divide-first" reasoning (N=47)		
	Yes	No	
		"Acceptance"	Other
Positive appraisal	**1** Recognition of rejection and agreement with it.	**6** Interpreted the story as acceptance and supported it. *There was an equity on questions asked. The teacher praised each student for their responses.* [24001_1261]	
Negative appraisal	**24** Recognition of rejection but disagreement with it. *It is very disappointing when teachers stopped the stduent's (sic!) eagerness to try other ways....* [24001_1263]	**0**	
Cannot be determined	**3** Recognition of rejection but no evaluation.		**13** Attending to unrelated aspects of the story.

How do open-ended categories match with cluster analysis?



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Conclusions

- Teachers view themselves as responsible for assuring that students have learned the canonical method for solving linear equations.
- At the same time there seems to be some openness to creativity and willingness to accept the divide-first solution, while making explicit its limitations as a general strategy.
- There is some evidence that there are other reasons that the teachers are uncomfortable with the actions of represented teacher in the scenario.